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Atiyah-Hitchin M-branes

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Abstract: We present new M2 and M5 brane solutions in M-theory based on transverse Atiyah-Hitchin space and other self-dual geometries. One novel feature of these solutions is that they have bolt-like fixed points yet still preserve 1/4 of the supersymmetry. All the solutions can be reduced down to ten dimensional intersecting brane configurations.

Keywords: M-Theory, D-branes, String Duality, Supergravity Models.

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Contents

1. Introduction

Fundamental M-theory in the low-energy limit is generally believed to be effectively described by $D = 11$ supergravity [1, 2, 3]. This suggests that brane solutions in the latter theory furnish classical soliton states of M-theory, motivating considerable interest in this subject. There is particular interest in supersymmetric p-brane solutions that saturate the BPS bound upon reduction to 10 dimensions. Some supersymmetric solutions of two or three orthogonally intersecting 2-branes and 5-branes in $D = 11$ supergravity were obtained some years ago [4], and more such solutions have since been found [5].

Recently interesting new supergravity solutions for localized D2/D6, D2/D4, NS5/D6 and NS5/D5 intersecting brane systems were obtained [6, 7, 8]. By lifting a D6 (D5 or D4)-brane to four-dimensional Taub-NUT/Bolt and Eguchi-Hanson geometries embedded in M-theory, these solutions were constructed by placing M2- and M5-branes in the Taub-NUT/Bolt and Eguchi-Hanson background geometries. The special feature of these constructions is that the solution is not restricted to be in the near core region of the D6 (D5 or D4)-brane.

Taub-NUT space is a special case of the Atiyah-Hitchin space and since the building blocks of M-theory are M2- and M5-branes, it is natural to investigate the possibility of placing M2- and M5-branes in the Atiyah-Hitchin background space. This is the subject of the present paper, in which we consider the embedding of Atiyah-Hitchin geometry in M-theory with an M2- or M5-brane. For all of the different solutions we obtain, 1/4 of the supersymmetry is preserved, and the metric has bolt-like fixed points (i.e. of maximal co-dimensionality). This is an interesting feature of all the solutions we obtain, quite distinct from all previously constructed M-brane solutions with bolt-like fixed points [7, 8], for which no supersymmetries are preserved. The difference arises as a result of the self-duality of the Atiyah-Hitchin metric compared to non-self-dual Taub-Bolt metrics. In the former case, self-duality preserves some supersymmetry while in the latter case, the lack of self-duality precludes any possible supersymmetry. We then compactify these solutions on a circle, obtaining the different fields of type-IIA string theory. Explicit calculation shows that in all cases the metric is asymptotically (locally) flat, though for some of our compactified solutions the type-IIA dilaton field diverges at infinity.

The Atiyah-Hitchin space is a part of the set of two monopole solutions of Bogomol'nyi equation. The moduli space of solutions is of the form

$$\mathbb{R}^3 \otimes \frac{S^1 \otimes \mathcal{M}}{\mathbb{Z}_2}$$

where the factor $\mathbb{R}^3 \otimes S^1$ describes the center of mass of two monopoles and a phase factor that is related to the total electric charge of the system. The interesting part of the moduli space is the four dimensional manifold $\mathcal{M}$, which has self-dual curvature. The self-duality comes from the hyper-Kähler property of the moduli space. Since $\mathbb{R}^3 \otimes S^1$ is flat and decouples from $\mathcal{M}$, the four dimensional manifold $\mathcal{M}$ should be hyper-Kähler, which is equivalent to a metric with self-dual curvature in four dimensions. The manifold $\mathcal{M}$ describes the separation of the two monopoles and their relative phase angle (or electric charges). A further aspect concerning $\mathcal{M}$ is that it should be SO(3) invariant, since two monopoles do exist in ordinary flat space; hence the metric on $\mathcal{M}$ can be expressed in terms of three functions of the monopole separation. Self-duality implies that these three functions obey a set of first-order ordinary differential equations.

In recent years, this space and its various generalizations were identified with the full quantum moduli space of $\mathcal{N} = 4$ supersymmetric gauge theories in three dimensions [9].

The outline of our paper is as follows. In section 2, we discuss briefly the field equations of supergravity, the M2- and M5-brane metrics and the Killing spinor equations. In section 3, we present the different M2-brane solutions that preserve 1/4 of the supersymmetry. We find type-IIA D2⊥D6(2) intersecting brane solutions upon dimensional reduction. In section 4 the alternative M2-brane solutions are presented. These solutions are obtained by continuation of the real separation constant into a pure imaginary separation constant. In section 5, we present different M5 solutions that preserve 1/4 of the supersymmetry; upon dimensional reduction at infinity, we find IIA NS5⊥D6(5) intersecting brane solutions.

JHEP10(2004)012

2. M2- and M5-branes and Kaluza-Klein reduction

The equations of motion for eleven dimensional supergravity when we have maximal symmetry (i.e. for which the expectation values of the fermion fields is zero), are [10]

$$R_{mn} - \frac{1}{2} g_{mn} R = \frac{1}{3}\left[F_{mpqr} F_n{}^{pqr} - \frac{1}{8} g_{mn} F_{pqrs} F^{pqrs}\right] \tag{2.1}$$

$$\nabla_m F^{mnpq} = -\frac{1}{576} \varepsilon^{m_1 \ldots m_8 npq} F_{m_1 \ldots m_4} F_{m_5 \ldots m_8} \tag{2.2}$$

where the indices $m, n, \ldots$ are 11-dimensional world space indices. For an M2-brane, we use the metric and four-form field strength

$$ds_{11}^2 = H(y,r)^{-2/3}\left(-dt^2 + dx_1^2 + dx_2^2\right) + H(y,r)^{1/3}\left(d\mathfrak{s}_4^2(y) + ds_4^2(r)\right) \tag{2.3}$$

and non-vanishing four-form field components

$$F_{tx_1x_2y} = -\frac{1}{2H^2}\frac{\partial H}{\partial y}\,, \qquad F_{tx_1x_2r} = -\frac{1}{2H^2}\frac{\partial H}{\partial r}\,. \tag{2.4}$$

and for an M5-brane, the metric and four-form field strength are

$$ds^2 = H(y,r)^{-1/3}\left(-dt^2 + dx_1^2 + \ldots + dx_5^2\right) + H(y,r)^{2/3}\left(dy^2 + ds_4^2(r)\right) \tag{2.5}$$

$$F_{m_1\ldots m_4} = \frac{\alpha}{2}\epsilon_{m_1\ldots m_5}\partial^{m_5}H \quad , \quad \alpha = \pm 1 \tag{2.6}$$

where $ds_4^2(y)$ and $ds_4^2(r)$ are two four-dimensional (euclideanized) metrics, depending on the non-compact coordinates y and r, respectively and the quantity $\alpha = \pm 1$, which corresponds to an M5 brane and an anti-M5 brane respectively. The general solution, where the transverse coordinates are given by a flat metric, admits a solution with 16 Killing spinors [11].

The 11D metric and four-form field strength can be easily reduced down to ten dimensions using the following equations

$$g_{mn} = \begin{bmatrix} e^{-2\Phi/3}\left(g_{\alpha\beta} + e^{2\Phi}C_\alpha C_\beta\right) & \nu e^{4\Phi/3}C_\alpha \\ \nu e^{4\Phi/3}C_\beta & \nu^2 e^{4\Phi/3} \end{bmatrix} \tag{2.7}$$

$$F_{(4)} = \mathcal{F}_{(4)} + \mathcal{H}_{(3)} \wedge dx_{10}\,. \tag{2.8}$$

Here ν is the winding number (the number of times the M5 brane wraps around the compactified dimensions) and x_{10} is the eleventh dimension, on which we compactify. We use hats in the above to differentiate the eleven-dimensional fields from the ten-dimensional ones that arise from compactification. $\mathcal{F}_{(4)}$ and $\mathcal{H}_{(3)}$ are the RR four-form and the NSNS three-form field strengths corresponding to $A_{\alpha\beta\gamma}$ and $B_{\alpha\beta}$.

The number of non-trivial solutions to the Killing spinor equation

$$\partial_m\epsilon + \frac{1}{4}\omega_{abm}\Gamma^{ab}\epsilon + \frac{1}{144}\Gamma_m{}^{npqr}F_{npqr}\epsilon - \frac{1}{18}\Gamma^{pqr}F_{mpqr}\epsilon = 0 \tag{2.9}$$

determine the amount of supersymmetry of the solution, where the ω's are the spin connection coefficients, and $\Gamma^{a_1\ldots a_n} = \Gamma^{[a_1}\ldots\Gamma^{a_n]}$. The indices $a, b, \ldots$ are 11 dimensional tangent space indices and the Γ^a matrices are the eleven dimensional equivalents of the four dimensional Dirac gamma matrices, and must satisfy the Clifford algebra

$$\left\{\Gamma^a, \Gamma^b\right\} = -2\eta^{ab}\,. \tag{2.10}$$

In ten dimensional type-IIA string theory, we can have D-branes or NS-branes. Dp-branes can carry either electric or magnetic charge with respect to the RR fields; the metric takes the form [11]

$$ds_{10}^2 = f^{-1/2}\left(-dt^2 + dx_1^2 + \cdots + dx_p^2\right) + f^{1/2}\left(dx_{p+1}^2 + \cdots + dx_9^2\right) \tag{2.11}$$

where the harmonic function f generally depends on the transverse coordinates.

An NS5-brane carries a magnetic two-form charge; the corresponding metric has the form

$$ds_{10}^2 = -dt^2 + dx_1^2 + \cdots + dx_5^2 + f\left(dx_6^2 + \cdots + dx_9^2\right)\,. \tag{2.12}$$

In what follows we will obtain a mixture of D-branes and NS-branes.

JHEP10(2004)012

3. Embedding Atiyah-Hitchin space in an M2-brane metric

The eleven dimensional M2-brane with an embedded transverse Atiyah-Hitchin space is given by the following metric

$$ds_{11}^2 = H(y,r)^{-2/3}\left(-dt^2 + dx_1^2 + dx_2^2\right) + H(y,r)^{1/3}\left(dy^2 + y^2 d\Omega_3^2 + ds_{AH}^2\right) \tag{3.1}$$

and non-vanishing four-form field components

$$F_{tx_1x_2y} = -\frac{1}{2H^2}\frac{\partial H}{\partial y}\,, \qquad F_{tx_1x_2r} = -\frac{1}{2H^2}\frac{\partial H}{\partial r}\,. \tag{3.2}$$

The Atiyah-Hitchin metric ds_{AH}^2 is given by the following manifestly SO(3) invariant form [12]

$$ds_{AH}^2 = f^2(r)dr^2 + a^2(r)\sigma_1^2 + b^2(r)\sigma_2^2 + c^2(r)\sigma_3^2 \tag{3.3}$$

with

$$\begin{aligned}
\sigma_1 &= -\sin\psi d\theta + \cos\psi \sin\theta d\phi \\
\sigma_2 &= \cos\psi d\theta + \sin\psi \sin\theta d\phi \\
\sigma_3 &= d\psi + \cos\theta d\phi
\end{aligned} \tag{3.4}$$

where σ_i are Maurer-Cartan one-forms with the property

$$d\sigma_i = \frac{1}{2}\varepsilon_{ijk}\sigma_j \wedge \sigma_k\,. \tag{3.5}$$

We note that the metric on the $\mathbb{R}^4$ (with a radial coordinate R and Euler angles (θ, ϕ, ψ) on an S^3) could be written in terms of Maurer-Cartan one-forms by

$$ds^2 = dR^2 + \frac{R^2}{4}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2)\,. \tag{3.6}$$

We also note that $\sigma_1^2 + \sigma_2^2$ is the standard metric of the round unit radius S^2 and $4(\sigma_1^2 + \sigma_2^2 + \sigma_3^2)$ gives the same for S^3. The metric (3.3) satisfies Einstein's equations provided

$$\begin{aligned}
a' &= f\frac{(b-c)^2 - a^2}{2bc} \\
b' &= f\frac{(c-a)^2 - b^2}{2ca} \\
c' &= f\frac{(a-b)^2 - c^2}{2ab}\,.
\end{aligned} \tag{3.7}$$

Choosing $f(r) = -b(r)/r$ the explicit expressions for the metric functions a, b and c are given by

$$\begin{aligned}
a(r) &= \sqrt{\frac{r\Upsilon\sin(\gamma)\{\frac{1-\cos(\gamma)}{2}r - \sin(\gamma)\Upsilon\}}{\Upsilon\sin(\gamma) + r\cos^2(\frac{\gamma}{2})}} \\
b(r) &= \sqrt{\frac{\{\Upsilon\sin(\gamma) - \frac{1-\cos\gamma}{2}r\}r\{-\Upsilon\sin(\gamma) - \frac{1+\cos\gamma}{2}r\}}{\Upsilon\sin(\gamma)}}
\end{aligned}$$

$$c(r) = -\sqrt{\frac{r\Upsilon\sin(\gamma)\{\frac{1+\cos(\gamma)}{2}r+\sin(\gamma)\Upsilon\}}{-\Upsilon\sin(\gamma)+\frac{1-\cos\gamma}{2}r}} \tag{3.8}$$

where

$$\Upsilon = \frac{2nE\{\sin(\frac{\gamma}{2})\}}{\sin(\gamma)} - \frac{nK\{\sin(\frac{\gamma}{2})\}\cos(\frac{\gamma}{2})}{\sin(\frac{\gamma}{2})} \tag{3.9}$$

and

$$K\left(\sin\left(\frac{\gamma}{2}\right)\right) = \frac{r}{2n}. \tag{3.10}$$

In the above equations, K and E are the elliptic integrals

$$K(k) = \int_0^1 \frac{dt}{\sqrt{1-t^2}\sqrt{1-k^2t^2}} = \int_0^{\pi/2}\frac{d\theta}{\sqrt{1-k^2\cos^2\theta}} \tag{3.11}$$

$$E(k) = \int_0^1 \frac{\sqrt{1-k^2t^2}dt}{\sqrt{1-t^2}} = \int_0^{\pi/2}\sqrt{1-k^2\cos^2\theta}d\theta \tag{3.12}$$

and the coordinate r ranges over the interval $[n\pi,\infty)$, which corresponds to $\gamma\in[0,\pi)$. The positive number n is a constant number with unit of length that is related to NUT charge of metric at infinity obtained from Atiyah-Hitchin metric.

In fact as $r\to\infty$, the metric (3.3) reduces to

$$ds^2_{AH} \to \left(1-\frac{2n}{r}\right)(dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2) + 4n^2\left(1-\frac{2n}{r}\right)^{-1}(d\psi+\cos\theta d\phi)^2 \tag{3.13}$$

which is the well known euclidean Taub-NUT metric with a negative NUT charge $N=-n$. One can compactify the M2-brane solution at infinity over the circle described by the coordinate ψ, which from equation (3.13) has radius $2n$.

The above metric is obtained from a consideration of the limiting behaviors of the functions a, b and c at large monopole separation that are given by

$$\begin{aligned} a(r) &= r\left(1-\frac{2n}{r}\right)^{1/2} + O(e^{-r/n}) \\ b(r) &= r\left(1-\frac{2n}{r}\right)^{1/2} + O(e^{-r/n}) \\ c(r) &= -2n\left(1-\frac{2n}{r}\right)^{-1/2} + O(e^{-r/n}). \end{aligned} \tag{3.14}$$

The metric (3.1) is a solution to the eleven dimensional supergravity equations provided $H(y,r)$ is a solution to the differential equation

$$r^2\frac{\partial^2 H}{\partial r^2} + r\left\{r\frac{(ac)'}{ac}+1\right\}\frac{\partial H}{\partial r} + b^2\left\{\frac{\partial^2 H}{\partial y^2}+\frac{3}{y}\frac{\partial H}{\partial y}\right\} = 0. \tag{3.15}$$

This equation is straightforwardly separable. Substituting

$$H(y,r) = 1 + Q_{M2}Y(y)R(r) \tag{3.16}$$

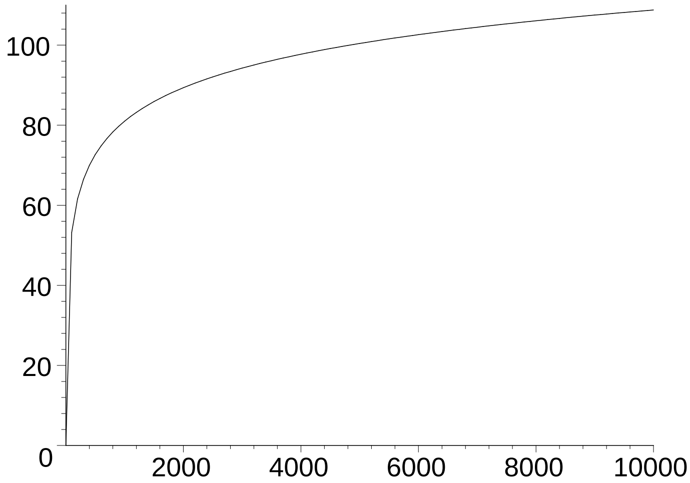

Figure 1: Numerical solution of the radial equation (3.18) for $R_k/10^7$ as a function of $\frac{1}{r-\pi}$, where we set $n = 1$ for simplicity. So for $r \approx \pi$, R diverges logarithmically as $\ln(\frac{1}{r-\pi})$. Note that the plot is only reliable for large $\frac{1}{r-\pi}$.

where Q_{M2} is the charge on the M2 brane, we arrive at two differential equations for $Y(y)$ and $R(r)$. The solution of the differential equation for $Y(y)$ is

$$Y(y) = \frac{J_1(ky)}{y} \tag{3.17}$$

which has a damped oscillating behavior at infinity. The differential equation for $R(r)$ is

$$acr^2 \frac{d^2 R_k(r)}{dr^2} + \left\{ acr - \frac{1}{2} r \left[(a-b)^2 + (b-c)^2 - a^2 - c^2 \right] \right\} \frac{dR_k(r)}{dr} - k^2 ab^2 c R_k(r) = 0 \tag{3.18}$$

where we have used the equations (3.7) and k is the separation constant. Although equation (3.18) does not have any analytic closed solution, we can solve it numerically. A typical numerical solution of (3.18) is given in figure 1, where for simplicity we set $n = 1$. The qualitative behaviour of the numerical solutions of equation (3.18) for other values of n is similar to figure 1, with the logarithmic divergence shifting to the point $r \simeq \pi n$.

The most interesting point is near $r \simeq \pi n$. Indeed, the plot in figure 1 is reliable only near $r \simeq \pi n$, corresponding to very large values on the horizontal axis. The divergence of the radial function at $r \simeq \pi n$ is given by

$$R_k(r) \simeq K_0(k(r - \pi n)) = -\ln\left(\frac{k}{2}\right) - \gamma - \ln(r - \pi n) + O\{(r - \pi n)^2\} \tag{3.19}$$

where K_0 is the modified Bessel function of the second kind and γ is the Euler-Mascheroni constant.

The behaviour of a, b and c near $r \simeq \pi n$ is

$$\begin{aligned} a(r) &= 2(r - \pi n) + O((r - \pi n)^2) \\ b(r) &= \pi n + \frac{1}{2}(r - \pi n) + O((r - \pi n)^2) \end{aligned}$$

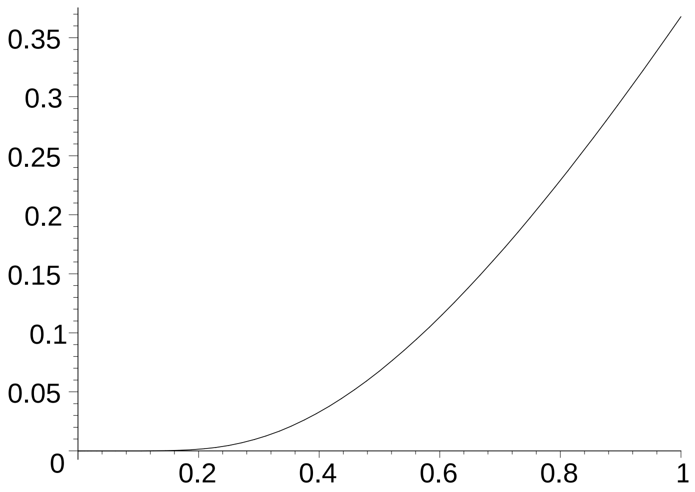

Figure 2: Behavior of the radial function R_k as a function of $1/r$, near infinity.

$$c(r) = -\pi n + \frac{1}{2}(r - \pi n) + O((r - \pi n)^2) \tag{3.20}$$

which indicates a bolt singularity at this point since $a(r) \simeq 0$. By using the SO(3) invariance of the metric, we can write the metric element (3.3) near the bolt location as

$$ds^2 = dr^2 + 4(r - \pi n)^2(d\widetilde{\psi} + \cos\widetilde{\theta} d\widetilde{\phi})^2 + \pi^2 n^2(d\widetilde{\theta} + \sin^2\widetilde{\theta} d\widetilde{\phi}) . \tag{3.21}$$

where $\widetilde{\psi}, \widetilde{\theta}$ and $\widetilde{\phi}$ are a new set of Euler angles related to ψ, θ, ϕ by

$$\mathcal{R}_1(\widetilde{\psi})\mathcal{R}_3(\widetilde{\theta})\mathcal{R}_1(\widetilde{\phi}) = \mathcal{R}_3(\psi)\mathcal{R}_2(\theta)\mathcal{R}_3(\phi) \tag{3.22}$$

in which $\mathcal{R}_i(\alpha)$ represents a rotation by α about the i th axis. Note that the last term in (3.21) is the induced metric on the two dimensional bolt.

To find the behaviour of the radial function $R_k(r)$ at large r, we use the relations (3.14); in this case we find

$$R_k(r) \simeq \frac{e^{-\frac{k^2}{|k|}r}}{r} \tag{3.23}$$

a plot of which is given in figure 2.

The final solution will be a superposition of all possible solutions and takes the form

$$H(y, r) = 1 + Q_{M2} \int_0^\infty dk p(k) \frac{J_1(ky)}{y} R_k(r) \tag{3.24}$$

where $p(k)$ is the measure function.

As $r \to \infty$ the Atiyah-Hitchin metric reduces to the $N = -n$ Taub-NUT metric. It is tempting to use this to fix the form of the measure function $p(k)$ via comparison to the M2-brane solution obtained by embedding a Taub-NUT space [6, 7]. However this is not correct; the derivation of the measure function for the Taub-NUT based M2-brane (with positive NUT charge $\mathcal{N}$) assumed that $r \ll \mathcal{N}$ [6, 7]. In the present case the Taub-NUT metric (3.13) is well defined only for $r \geq 2n$.

To fix $p(k)$ we compare the relation (3.24) to that of the metric function of an M2-brane in a transverse flat metric $\mathbb{R}^4 \otimes \mathbb{R}^2 \otimes S^2$, obtained by looking at the near bolt limit. At $r \simeq \pi n$, for a fixed point on the bolt, the metric reduces to the metric of $\mathbb{R}^2$, that is $d\widetilde{r}^2 + 4\widetilde{r}^2 d\widetilde{\psi}^2$ where $\widetilde{r} = r - n\pi$. In this case the radial function is given by (3.19); to obtain reduction of the metric function (3.24) to $1 + \frac{Q_{M2}}{\mathcal{R}^6}$ where $\mathcal{R} = \sqrt{y^2 + \widetilde{r}^2}$, we must fix the measure function to be $p(k) \propto k^4$. Absorbing the constant into the M2-brane charge yields

$$H_{AH}(y,r) = 1 + Q_{M2} \int_0^\infty dk k^4 \frac{J_1(ky)}{y} R_k(r) \tag{3.25}$$

as the metric function of the M2-brane solution (3.1).

Since $\partial/\partial\psi$ is not a Killing vector (except for $a(r) = b(r)$) we cannot use the reduction relations (2.7) and (2.8) to find 10D type-IIA fields explicitly. However, we note that at large r, we have $a(r) = b(r)$ up to terms of order of $e^{-r/n}$. Hence the reduction relations (2.7) and (2.8) at large r yield the explicit fields in 10D that describe a D2⊥D6(2) system which preserves 1/4 of the supersymmetry [7]. So although we cannot explicitly write the 10D brane system fields corresponding to the M2-brane solution (3.1), the asymptotic behaviour of the system is given by the known fields of D2⊥D6(2) system which preserves 1/4 of the supersymmetry [7].

The second possible M2-brane solution is given by

$$ds_{11}^2 = H(y,r)^{-2/3}\left(-dt^2 + dx_1^2 + dx_2^2\right) + H(y,r)^{1/3}\left(ds_{TN}^2 + ds_{AH}^2\right) \tag{3.26}$$

where

$$ds_{TN}^2 = f(y)\left(dy^2 + y^2(d\alpha^2 + \sin^2(\alpha) d\beta^2)\right) + \left(\frac{(4\mathbf{n})^2}{f(y)}\right)\left(d\sigma + \frac{1}{2}\cos(\alpha) d\beta\right)^2 \tag{3.27}$$

$$f(y) = \left(1 + \frac{2\mathbf{n}}{y}\right). \tag{3.28}$$

In this case, after separation of variables by the relation (3.16), we find the same differential equation for $R(r)$ as given in equation (3.18), and the solution of the differential equation for $Y(y)$ which has a damped oscillating behavior at infinity up to a constant, is

$$Y(y) = \frac{(-i)\mathcal{W}_M(-ik\mathbf{n}, 1/2, 2iky)}{2ky} \tag{3.29}$$

where $\mathcal{W}_M$ is the Whittaker function. The final general solution will be a superposition of all possible solutions in the form

$$H_{TN\,\otimes\,AH}(y,r) = 1 + Q_{M2} \int_0^\infty dk q(k) \frac{(-i)\mathcal{W}_M(-ik\mathbf{n}, 1/2, 2iky)}{2ky} R_k(r) \tag{3.30}$$

where as before $q(k)$ can be determined by looking at some near horizon/bolt limit. For $r \simeq \pi n$ and $y \ll \mathbf{n}$, the transverse metric reduces to $\mathbb{R}^4 \otimes \mathbb{R}^2 \otimes S^2$ since the Taub-NUT metric (3.27) reduces to $\mathbb{R}^4$ with line element $d\mathfrak{z}^2 + \mathfrak{z}^2 d\Omega_3^2$, where $\mathfrak{z} = 2\sqrt{2\mathbf{n}y}$. The transverse radial distance to the bolt is given by $\mathcal{R} = \sqrt{\mathfrak{z}^2 + \widetilde{r}^2}$. Comparing the metric function (3.30) with that of an M2-brane in transverse flat space, we can fix the measure function to be $q(k) \propto k^5$. Absorbing the constant into the M2-brane charge, we obtain

$$H_{TN \otimes AH}(y,r) = 1 - iQ_{M2} \int_0^\infty dk k^4 \frac{\mathcal{W}_M(-ik\mathbf{n}, 1/2, 2iky)}{y} R_k(r) \tag{3.31}$$

as the metric function of M2-brane solution (3.26). Note that the Whittaker function in the integrand is pure imaginary, yielding a real-valued $H_{TN \otimes AH}(y,r)$.

Compactifying over the circle parametrized by σ (and noting that $\partial/\partial\sigma$ is a Killing vector) we find the NSNS fields

$$\begin{aligned} \Phi &= \frac{3}{4} \ln \left(\frac{H^{1/3}}{f} \right) \\ B_{\mu\nu} &= 0 \end{aligned} \tag{3.32}$$

and RR fields

$$\begin{aligned} C_\beta &= 2\mathbf{n} \cos(\alpha) \\ A_{tx_1x_2} &= H^{-1} . \end{aligned} \tag{3.33}$$

The metric in ten dimensions will be given by

$$\begin{aligned} ds_{10}^2 = {} & H_{TN \otimes AH}(y,r)^{-1/2} f^{-1/2} \left(-dt^2 + dx_1^2 + dx_2^2\right) + H_{TN \otimes AH}(y,r)^{1/2} f^{-1/2} ds_{AH}^2 + \\ & + H_{TN \otimes AH}(y,r)^{1/2} f^{1/2} \left(dy^2 + y^2(d\alpha^2 + \sin^2(\alpha) d\beta^2)\right) . \end{aligned} \tag{3.34}$$

This represents a D2⊥D6(2) system that preserves 1/4 of the supersymmetry. We have explicitly checked that the above 10-dimensional metric, with the given dilaton, one and three forms, is a solution to the 10-dimensional supergravity equations of motion.

The third possible M2-brane solution is given by

$$ds_{11}^2 = H(y,r)^{-2/3} \left(-dt^2 + dx_1^2 + dx_2^2\right) + H(y,r)^{1/3} \left(ds_{EH}^2 + ds_{AH}^2\right) \tag{3.35}$$

where

$$ds_{EH}^2 = \frac{y^2}{4g(y)} \left[d\sigma + \cos(\alpha) d\beta\right]^2 + g(y) dy^2 + \frac{y^2}{4} \left(d\alpha^2 + \sin^2(\alpha) d\beta^2\right) \tag{3.36}$$

$$g(r) = \left(1 - \frac{a^4}{y^4}\right)^{-1} . \tag{3.37}$$

is the Eguchi-Hanson metric ds_{EH}^2. In this case, after separation of variables by the relation (3.16), we find the same differential equation for $R(r)$ as given in equation (3.18), and the differential equation for $Y(y)$ is

$$y(y^4 - a^4) Y_k''(y) + (3y^4 + a^4) Y_k'(y) + k^2 y^5 Y_k(y) = 0 . \tag{3.38}$$

While an analytic closed solution for the differential equation (3.38) is not available, the numerical solution shows that it has a damped oscillating behaviour at infinity which

JHEP10(2004)012

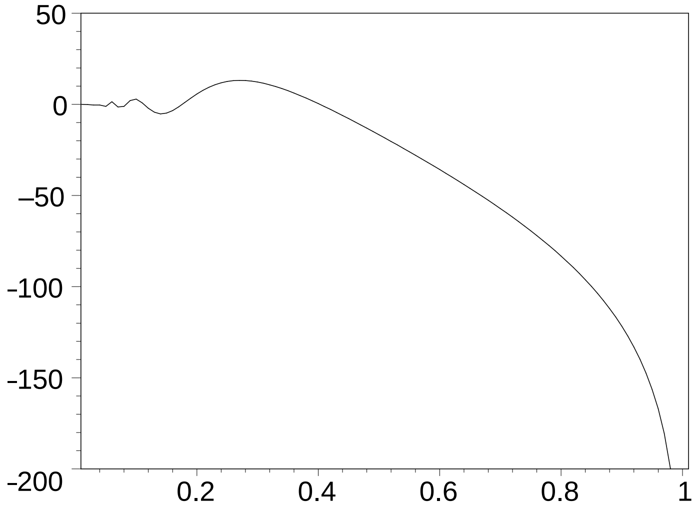

Figure 3: Numerical solution of equation (3.38) for $Y_k/10^5$ as a function of $1/y$ for non-zero separation constant k. The Eguchi-Hanson parameter a is set to one and so for $y \approx a$, the function Y_k diverges and for $y \approx \infty$, it vanishes.

diverges at $y \simeq a$. A typical numerical solution of $Y_k(y)$ is given in figure 3. The general solution will be a superposition of all possible solutions in the form

$$H_{EH \,\otimes\, AH}(y,r) = 1 + Q_{M2} \int_0^\infty dk r(k) Y_k(y) R_k(r) \tag{3.39}$$

and we determine $r(k)$ by looking at a near horizon/bolt limit. At $r \simeq \pi n$ and $y \simeq a$, the transverse metric reduces to $\mathbb{R}^2 \otimes S^2 \otimes \mathbb{R}^2 \otimes S^2$, since the Eguchi-Hanson metric (3.36) for $y = a(1+\epsilon^2)$, where $\epsilon \ll 1$, reduces to

$$ds^2_{EH} \approx \mathbf{z}^2 d\sigma^2 + d\mathbf{z}^2 + \frac{a^2}{4}\left(d\alpha^2 + \sin^2(\alpha) d\beta^2\right) \tag{3.40}$$

which is the metric of $\mathbb{R}^2 \otimes S^2$ and $\mathbf{z} = a\epsilon$. In this limit and for small a, the differential equation (3.38) has the real solution

$$Y(\widehat{y}) = -i\frac{I_1(ik\widehat{y})}{\widehat{y}} \tag{3.41}$$

that vanishes at infinity where $\widehat{y} = a\epsilon^2$. The transverse radial distance to bolt is given by $\mathcal{R} = \sqrt{\mathbf{z}^2 + \frac{a^2}{4} + \widetilde{r}^2}$. Comparing the metric function (3.39) with that of an M2-brane in transverse flat space, we fix the measure function $r(k) \propto k^5$, yielding

$$H_{EH \,\otimes\, AH}(y,r) = 1 + Q_{M2} \int_0^\infty dk k^5 Y_k(y) R_k(r) \tag{3.42}$$

for the metric function of the M2-brane solution (3.35), where we have absorbed a constant into the M2-brane charge.

In this case, by compactification along the σ direction of Eguchi-Hanson metric, we find the NSNS fields

$$\begin{aligned}\Phi &= \frac{3}{4}\ln\left\{\frac{H^{1/3}w^2}{4g}\right\}\\ B_{\mu\nu} &= 0\end{aligned} \tag{3.43}$$

and RR fields

$$\begin{aligned}C_\beta &= a\cos(\alpha)\\ A_{tx_1x_2} &= \frac{1}{H}\end{aligned} \tag{3.44}$$

and metric

$$\begin{aligned}ds_{10}^2 = \frac{w}{2}\Big\{&H^{-1/2}g^{-1/2}\left(-dt^2+dx_1^2+dx_2^2\right)+H^{1/2}g^{-1/2}ds_{AH}^2+\\ &+H^{1/2}g^{1/2}a^2\left\{dw^2+\frac{w^2}{4g}\left(d\alpha^2+\sin^2\alpha d\beta^2\right)\right\}\Big\}\end{aligned} \tag{3.45}$$

where $w=y/a$. The metric describes an intersecting D2/D6 system where D2 is localized along the world-volume of the D6-brane and the world-volume of the D6 brane transverse to D2 is just Atiyah-Hitchin space. We note that in the large w limit, the metric (3.45), reduces to the metric

$$ds_{10}^2 = \frac{w}{2}\left\{-dt^2+dx_1^2+dx_2^2+ds_{AH}^2+a^2\left(dw^2+\frac{w^2}{4}(d\alpha^2+\sin^2\alpha d\beta^2)\right)\right\} \tag{3.46}$$

which is again a 10D locally asymptotically flat metric with Kretchmann invariant

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{A(a,r)}{w^2}+\frac{224}{w^6a^4} \tag{3.47}$$

which vanishes at large w. $A(a,r)$ is a complicated function of the Eguchi-Hanson parameter a and Atiyah-Hitchin metric functions $a(r), b(r)$ and $c(r)$. All the components of the Riemann tensor in the orthonormal basis approach zero at $w\to\infty$.

Since $\partial/\partial\psi$ is a Killing vector we can further reduce the metrics (3.34) and (3.45) along the ψ direction of the Atiyah-Hitchin space at large r where $a(r)=b(r)$ (up to terms of order of $e^{-r/n}$). However the result of this compactification is not the same as the reduction of the M-theory solution over a torus, which is compactified type-IIB theory. To get the compactified type-IIB theory, we must T-dualize the metrics (3.34) and (3.45) first and then compactify the resultant type-IIB solutions along the ψ direction of the Atiyah-Hitchin space.

Finally, the fourth possible M2-brane solution is given by

$$ds_{11}^2 = H(r_1,r_2)^{-2/3}\left(-dt^2+dx_1^2+dx_2^2\right)+H(r_1,r_2)^{1/3}\left(ds_{AH_1}^2+ds_{AH_2}^2\right) \tag{3.48}$$

where $ds_{AH_1}^2$ and $ds_{AH_2}^2$ are given by two copies of (3.3) with coordinate systems (r_1, θ_1, ϕ_1, ψ_1) and $(r_2,\theta_2,\phi_2,\psi_2)$, respectively. In this case, after separation of variables by the relation

$$H(r_1,r_2) = 1+Q_{M2}\widetilde{R}(r_1)R(r_2) \tag{3.49}$$

we find two differential equations

$$a_1c_1r_1^2\frac{d^2\widetilde{R}_k(r_1)}{dr_1^2}+\left\{a_1c_1r_1-\frac{1}{2}r_1[(a_1-b_1)^2+(b_1-c_1)^2-a_1^2-c_1^2]\right\}\times$$

$$\times\frac{d\widetilde{R}_k(r_1)}{dr_1}+k^2a_1b_1^2c_1\widetilde{R}_k(r_1)=0 \qquad (3.50)$$

$$a_2c_2r_2^2\frac{d^2R_k(r_2)}{dr_2^2}+\left\{a_2c_2r_2-\frac{1}{2}r_2[(a_2-b_2)^2+(b_2-c_2)^2-a_2^2-c_2^2]\right\}\times$$

$$\times\frac{dR_k(r_2)}{dr_2}-k^2a_2b_2^2c_2R_k(r_2)=0\,. \qquad (3.51)$$

We note the differential equation for $R_k(r_2)$ is the same as equation (3.18), that its typical solution is presented in figures 1 and 2 for near bolt and near infinity regions. As before, an analytic closed solution for the first differential equation (3.50) is not available. However the numerical solution (presented in figures 4 and 5 in section 4) shows that it has diverging behaviour at $r_1\simeq\pi n_1$ and damped oscillating behaviour at infinity. In fact we note later that for $r_1\simeq\pi n_1$ (the only reliable region in figure 4), the function $\widetilde{R}_k(r_1)$ has a logarithmic divergence.

The general solution will be a superposition of all possible solutions in the form

$$H_{AH\,\otimes\,AH}(r_1,r_2)=1+Q_{M2}\int_0^\infty dks(k)\widetilde{R}_k(r_1)R_k(r_2) \qquad (3.52)$$

where $s(k)\propto k^5$ following previous near-horizon/bolt arguments and comparing the metric function (3.52) with that of M2-brane in transverse flat space . In this case as $r_1\simeq\pi n_1$ and $r_2\simeq\pi n_2$, the transverse metric in (3.48) reduces to $\mathbb{R}^4\otimes\mathbb{R}^4$ with line element $d\mathfrak{z}_1^2+d\mathfrak{z}_2^2+\mathfrak{z}_1^2d\Omega_3^2+\mathfrak{z}_2^2d\Omega_3'^2$, where $\mathfrak{z}_i=2\sqrt{2n_ir_i}$ and the transverse radial distance to the bolt is given by $\mathcal{R}=\sqrt{\mathfrak{z}_1^2+\mathfrak{z}_2^2}$. We find

$$H_{AH\,\otimes\,AH}(r_1,r_2)=1+Q_{M2}\int_0^\infty dkk^5\widetilde{R}_k(r_1)R_k(r_2) \qquad (3.53)$$

where we have absorbed additional constants into the M2-brane charge.

Since $\partial/\partial\psi_1$ and $\partial/\partial\psi_2$ are not Killing vectors except for $a_1(r_1)=b_1(r_1)$ and $a_2(r_2)=b_2(r_2)$, we cannot use the reduction relations (2.7) and (2.8) to find 10D type-IIA fields explicitly. However, we note that at large r_1 (or at large r_2), we have $a_1(r_1)=b_1(r_1)$ (or $a_2(r_2)=b_2(r_2)$) up to terms of order of e^{-r_1/n_1} (or e^{-r_2/n_2}). Consequently, although we cannot explicitly write the 10D brane system fields corresponding to the M2-brane solution (3.1), the reduction relations (2.7) and (2.8) at large r_1 (or at large r_2) yield the explicit fields in 10D that describe a D2$\perp$ D6(2) system that preserves 1/4 of the supersymmetry [7].

To summarize, all M2-brane solutions with an Atiyah-Hitchin space in the transverse geometry preserve 1/4 of the supersymmetry even though Atiyah-Hitchin space has a bolt-like fixed point at $r=\pi n$. This behaviour is completely different from what was observed in [7], where the only M2-brane solutions preserving any supersymmetry had NUT-like fixed points (i.e. of less than maximal co-dimensionality). M2-brane solutions with transverse

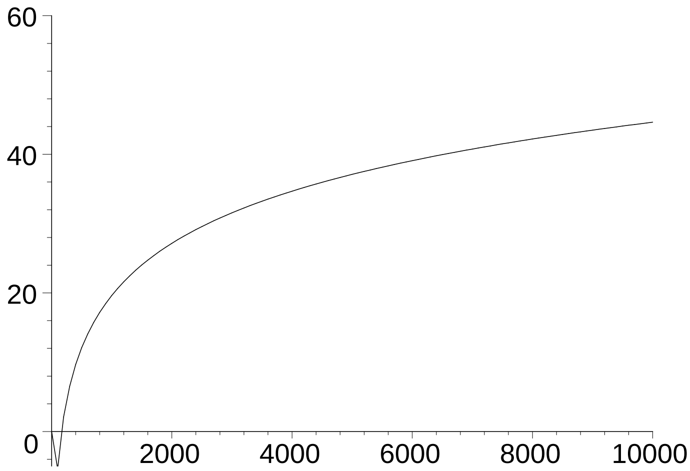

Figure 4: Numerical solution of the radial equation (4.2) for $\widetilde{R}_{\widetilde{k}}/10^6$ as a function of $\frac{1}{r-\pi}$, where we set $n = 1$ for simplicity. So for $r \approx \pi$, $\widetilde{R}$ diverges logarithmically by $\ln(\frac{1}{r-\pi})$. Note that the plot is reliable only for large $\frac{1}{r-\pi}$.

Taub-Bolt spaces of various dimensionalities were not supersymmetric. Unlike these cases, the four-dimensional Atiyah-Hitchin space is self-dual hyper-Kähler, thereby preserving some supersymmetry in the associated M2-brane solutions.

4. A second set of M2-brane solutions

A different set of M2-brane solutions can be obtained by reversing the sign of the separation constant k^2 in the separated differential equations for $Y(y)$ and $R(r)$. As an example, by taking $k \to i\widetilde{k}$ in the separable equations of Atiyah-Hitchin case, we find the solution of the differential equation for $\widetilde{Y}(y)$ as

$$\widetilde{Y}(y) = \frac{K_1(\widetilde{k}y)}{y} \tag{4.1}$$

where K_1 is the modified Bessel function, diverging at $y = 0$ and vanishing at infinity. The differential equation for $\widetilde{R}(r)$ is given by

$$acr^2 \frac{d^2 \widetilde{R}_{\widetilde{k}}(r)}{dr^2} + \left\{ acr - \frac{1}{2} r[(a-b)^2 + (b-c)^2 - a^2 - c^2] \right\} \frac{d\widetilde{R}_{\widetilde{k}}(r)}{dr} + \widetilde{k}^2 ab^2 c \widetilde{R}_{\widetilde{k}}(r) = 0\,. \tag{4.2}$$

Although the above equation does not have any analytic closed solution, we can solve it numerically. Typical solutions are presented in figures 4 and 5 for $r \simeq \pi n$ and large r regions respectively, where we set $n = 1$. We note that for $r \simeq \pi n$ (the only reliable region in figure 4), the function $\widetilde{R}_{\widetilde{k}}(r)$ also has a logarithmic divergence given by

$$\widetilde{R}_{\widetilde{k}}(r) \simeq -Y_0(\widetilde{k}(r - \pi n)) = -\frac{2}{\pi} \left\{ \ln\left(\frac{\widetilde{k}}{2}\right) + \gamma + \ln(r - \pi n) \right\} + O\{(r - \pi n)^2\} \tag{4.3}$$

where Y_0 is the Bessel function of the second kind.

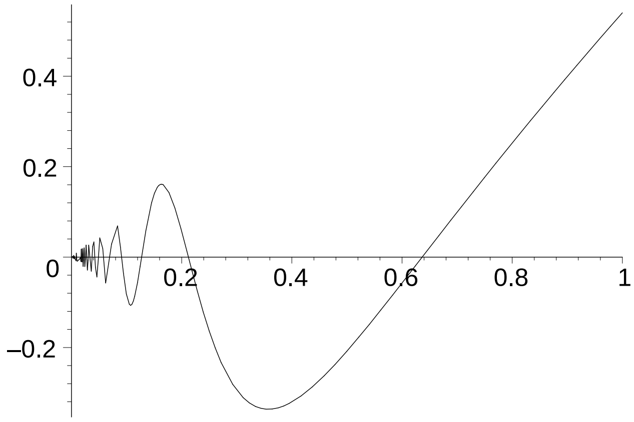

Figure 5: Behavior of the radial function $\widetilde{R}_{\widetilde{k}}(r)$ as a function of $1/r$, near infinity.

Figure 5 shows the behaviour of the radial function $\widetilde{R}_{\widetilde{k}}(r)$ at large r, given by

$$\widetilde{R}_{\widetilde{k}}(r) \simeq \frac{\cos(\widetilde{k}r)}{r} \tag{4.4}$$

which is obtained by using the relations (3.14). The final general solution will be a superposition of all possible solutions and has the form

$$\widetilde{H}(y,r) = 1 + Q_{M2}\int_0^\infty d\widetilde{k}\widetilde{p}(\widetilde{k})\frac{K_1(\widetilde{k}y)}{y}\widetilde{R}_{\widetilde{k}}(r) \tag{4.5}$$

where $\widetilde{p}(\widetilde{k})$ can be computed by comparing the relation (4.5) to that of a metric function of an M2-brane in a transverse flat metric $\mathbb{R}^4 \otimes \mathbb{R}^2 \otimes S^2$, obtained by looking at the near bolt limit. We obtain

$$\widetilde{H}_{AH}(y,r) = 1 + Q_{M2}\int_0^\infty d\widetilde{k}\widetilde{k}^4\frac{K_1(\widetilde{k}y)}{y}\widetilde{R}_{\widetilde{k}}(r) \tag{4.6}$$

as the second M2-brane solution (3.1), absorbing a possible constant into the charge Q_{M2}.

The other two alternative solutions for the Taub-NUT $\otimes$ Atiyah-Hitchin and Eguchi-Hanson $\otimes$ Atiyah-Hitchin could be derived easily similar to the above case and so we do not present them here. In the Atiyah-Hitchin $\otimes$ Atiyah-Hitchin case with metric function (3.53), the transformation $k \to i\widetilde{k}$ in (3.50) and (3.51), merely interchanges $\widetilde{R}_k(r_1) \to R_{\widetilde{k}}(r_1)$ and $R_k(r_2) \to \widetilde{R}_{\widetilde{k}}(r_2)$ and so yields no new solution.

5. Embedding Atiyah-Hitchin space in an M5-brane metric

The eleven dimensional M5-brane metric with an embedded Atiyah-Hitchin metric has the following form

$$ds_{11}^2 = H(y,r)^{-1/3}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2) + H(y,r)^{2/3}(dy^2 + ds_{AH}^2) \tag{5.1}$$

with field strength components

$$F_{\psi\theta\phi y} = \frac{\alpha}{2}\sin(\theta) rac\frac{\partial H}{\partial r}$$
$$F_{\psi\theta\phi r} = -\frac{\alpha}{2r}\sin(\theta) ab^2 c\frac{\partial H}{\partial y}\,. \tag{5.2}$$

We consider the M5-brane which corresponds to $\alpha = +1$; the $\alpha = -1$ case corresponds to an anti-M5 brane.

The metric (5.1) is a solution to the eleven dimensional supergravity equations provided $H(y, r)$ is a solution to the differential equation

$$r^2\frac{\partial^2 H}{\partial r^2} + r\left\{1 + r\left(\frac{a'}{a} + \frac{c'}{c}\right)\right\}\frac{\partial H}{\partial r} + b^2\frac{\partial^2 H}{\partial y^2} = 0\,. \tag{5.3}$$

This equation is straightforwardly separable. Substituting

$$H(y,r) = 1 + Q_{M5}Y(y)R(r) \tag{5.4}$$

where Q_{M5} is the charge on the M5-brane. The solution of the differential equation for $Y(y)$ is

$$Y(y) = \cos(ky + \varsigma) \tag{5.5}$$

and the differential equation for $R(r)$ is given by the equation (3.18). The numerical solution of this equation near $r \simeq \pi n$, is again given by figure 1; the final solution is a superposition of all possible solutions

$$H(y,r) = 1 + Q_{M5}\int_0^\infty dk h(k)\cos(ky)R_k(r) \tag{5.6}$$

where $h(k)$ is the measure function.

To fix the measure function $h(k)$ we compare the relation (5.6) to that of a metric function of M5-brane in transverse flat metric $\mathbb{R}^4 \otimes \mathbb{R}^2 \otimes S^2$, obtained by looking at the near bolt limit $r \simeq \pi n$. In this case the radial function in (5.6) reduces to (3.19). Hence for reduction of the metric function (5.6) to $1 + \frac{Q_{M5}}{\mathcal{R}^3}$ where $\mathcal{R} = \sqrt{y^2 + \widetilde{r}^2}$ and $\widetilde{r} = r - \pi n$, we must fix the measure function to be $h(k) \propto k^2$, giving

$$H_{AH}(y,r) = 1 + Q_{M5}\int_0^\infty dk k^2\cos(ky)R_k(r) \tag{5.7}$$

as the metric function of M5-brane solution (5.1), where we absorb the constant into the M5-brane charge. We note that in equation (5.7), $R_k(r)$ for $r \simeq \pi n$ approaches the numerical solution, presented in figure 1 and for large r, is given by the limit of the radial function in (3.18) or equivalently by (3.23).

As with the Atiyah-Hitchin-based M2 solution we can dimensionally reduce our M5 solution to find 10D type-IIA fields at large r, since $\partial/\partial\psi$ is a Killing vector and $a(r) = b(r)$ up to terms of order of $e^{-r/n}$. The resulting fields describe an NS5$\perp$D6(5) system which preserves 1/4 of the supersymmetry. We expect in the decoupling limit of our solution that is in the limit of vanishing string coupling, the theory on the worldvolume of the NS5-branes is a type-IIA little string theory [8].

JHEP10(2004)012

A different M5-brane solution can be obtained by reversing the sign of the separation constant k^2 in the separated differential equations obtained from (5.3). In this case, by taking $k \to i\widetilde{k}$, we find another solution in the form of

$$\widetilde{H}_{AH}(y,r) = 1 + Q_{M5} \int_0^\infty d\widetilde{k}\widetilde{k}^2 e^{-\widetilde{k}y} \widetilde{R}_{\widetilde{k}}(r) \tag{5.8}$$

where numerical plot of the function $\widetilde{R}_{\widetilde{k}}$ is given in figures 4 and 5 for $r \simeq n\pi$ and large r regions, respectively. Although this is formally a solution, the integral in (5.8) is not convergent for all values of y. To make the integral convergent for $y < 0$, one can replace $e^{-\widetilde{k}y}$ by $e^{-\widetilde{k}|y|}$, but only at the price of introducing a source term at $y = 0$ in the corresponding Laplace equation for $\widetilde{H}_{AH}(y,r)$.

As before, we do not have a representation of 10D fields since $\partial/\partial\psi$ is not a Killing vector except for $a(r) = b(r)$. However, we note that at large r, we have $a(r) = b(r)$ up to terms of order of $e^{-r/n}$ and in this case, by using the reduction relations (2.7) and (2.8), the explicit fields in 10D describe an NS5⊥D6(5) system that preserves 1/4 of the supersymmetry [8]. So although the 10D brane system fields corresponding to M5-brane solution (5.1) with metric functions (5.7) or (5.8) cannot be written explicitly, the asymptotic behavior of the system is given by the known fields of the NS5⊥D6(5) system that preserves 1/4 of the supersymmetry. As the last case, we expect in the decoupling limit of solution corresponding to (5.8), the theory on the worldvolume of the NS5-branes is a type-IIA little string theory [8].

We see again that the M5-brane solution (5.7) with an Atiyah-Hitchin space in the transverse geometry preserves 1/4 of the supersymmetry even though the Atiyah-Hitchin space has a bolt-like fixed point at $r = n\pi$. This behaviour is completely different from that found in [8], where the only M5-brane solutions preserving any supersymmetry had NUT-like fixed points (i.e. of less than maximal co-dimensionality). The M5-brane solutions with four dimensional transverse Taub-Bolt space were not supersymmetric. Unlike these cases, the four-dimensional Atiyah-Hitchin space is self-dual hyper-Kähler, thereby preserving some supersymmetry in the associated M5-brane solutions.

6. Conclusion

By embedding Atiyah-Hitchin space into M-theory, we have found new classes of 2-brane and 5-brane solutions to $D = 11$ supergravity. These exact solutions are new M2- and M5-brane metrics with metric functions (3.25), (3.31), (3.42), (3.53), (4.6), (5.7) and (5.8) — these are the main results of this paper. The common feature of both solutions is that the brane function is a convolution of an exponentially decaying 'radial' function (for both branes) with a damped oscillating one. The 'radial' function vanishes far from the branes and diverges logarithmically near the brane core. The same logarithmic divergence near the brane happens in embedding of Eguchi-Hanson metric in M-theory where the divergence is milder than $1/r$, as in the case of embedding Taub-NUT space. Indeed, all of these properties of our solutions are similar to those previously obtained [7, 8] for the embedding of Eguchi-Hanson and Taub-NUT spaces.

JHEP10(2004)012

However our solutions have a feature that is quite distinct from these predecessors: they are bolt solutions (i.e. solutions whose fixed points in the transverse space have maximal dimensionality) that preserve 1/4 of the supersymmetry due to the self-dual hyper-Kähler character of the Atiyah-Hitchin metric. This is in contrast to earlier brane solutions of this type [7, 8], for which supersymmetry could only be preserved for NUT-like transverse metrics; their bolt counterparts did not preserve any supersymmetry.

Dimensional reduction of the M2 solutions to ten dimensions gives us intersecting IIA D2/D6 configurations that preserve 1/4 of the supersymmetry. For the M5 solutions, dimensional reduction yields IIA NS5/D6 brane systems overlapping in five directions.

In the standard case, the system of N_5 NS5-branes located at N_6 D6-branes can be obtained by dimensional reduction of N_5N_6 coinciding images of M5-branes in the flat transverse geometry. In this case, the worldvolume theory (the little string theory) of the IIA NS5-branes, in the absence of D6-branes, is a non-local non-gravitational six dimensional theory [13]. This theory has (2,0) supersymmetry (four supercharges in the 4 representation of Lorentz symmetry $\mathrm{Spin}(5,1)$) and an R-symmetry Spin(4) remnant of the original ten dimensional Lorentz symmetry. The presence of the D6-branes breaks the supersymmetry down to (1,0), with eight supersymmetries. Since we found that our solutions preserve 1/4 of the supersymmetry, we expect that the theory on NS5-branes is a new little string theory.

We note that in the limit of large r, where $a(r) = b(r)$, the decoupling limits of M2 and M5 Atiyah-Hitchin based brane solutions are qualitatively the same as what was found in references [6, 7, 8] for corresponding Taub-NUT based solutions. Although the asymptotic holographic duals of the decoupled theories are known, it would therefore be interesting to find the complete structures of the holographic duals of the decoupled theories. We leave this for future study.

We note that applying T-duality on our M2 and M5 solutions generates new IIB configurations. For example, by T-dualization along any spatial directions of the NS5 brane, type-IIA NS5$\perp$D6(5) system changes to type-IIB NS5$\perp$D5(4) brane configuration, overlapping in four directions.

The worldvolume theory of the IIB NS5-branes, in the absence of D5-branes, is a little string theory with (1,1) supersymmetry. The presence of the D5-brane, which has one transverse direction relative to NS5 worldvolume, breaks the supersymmetry down to eight supersymmetries. This is in good agreement with the number of supersymmetries in 10D IIB theory: T-duality preserves the number of original IIA supersymmetries, that is eight.

Moreover we conclude that the new IIA and IIB little string theories are T-dual: the actual six dimensional T-duality is the remnant of the original 10D T-duality after toroidal compactification.

Acknowledgments

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